**MATEMATYKA UBEZPIECZENIOWA**

POMOCNICZE WZORY 2

*Ubezpieczenia płatne w chwili śmierci.*

$$\bar{A}_x = \int_0^\infty v^t {}_tp_x \mu_{x+t} dt.$$

$$\bar{A}^1_{x:\overline{n}|} = \int_0^\infty v^t {}_tp_x \mu_{x+t} dt.$$

$$\bar{A}_{x:\overset{1}{\overline{n}|}} = v^n {}_np_x.$$

$$\bar{A}_{x:\overline{n}|} = \bar{A}^1_{x:\overline{n}|} + \bar{A}_{x:\overset{1}{\overline{n}|}}.$$

$${}_{m|}\bar{A}_x = {}_mp_x v^m \bar{A}_{x+m}.$$

*Ubezpieczenia płatne na koniec roku śmierci.*

$$A_x = \sum_{k=0}^{\infty} v^{k+1} {}_kp_x q_{x+k}.$$

$$A^1_{x:\overline{n}|} = \sum_{k=0}^{n-1} v^{k+1} {}_kp_x q_{x+k}.$$

$$A_{x:\overset{1}{\overline{n}|}} = v^n {}_np_x.$$

$$A_{x:\overline{n}|} = A^1_{x:\overline{n}|} + A_{x:\overset{1}{\overline{n}|}},$$

$${}_{m|}A_x = {}_mp_x v^m A_{x+m}.$$

$$\bar{A}_x = \frac{i}{\delta} A_x.$$

$$\bar{A}^1_{x:\overline{n}|} = \frac{i}{\delta} A^1_{x:\overline{n}|}.$$

$$A^{(m)}_x = \frac{i}{i^{(m)}} A_x.$$

$$(IA)_x = \sum_{k=0}^{\infty} (k+1) v^{k+1} {}_kp_x q_{x+k}.$$

$$(IA)_{x:\overset{1}{\overline{n}|}} = \sum_{k=0}^{n-1} (k+1) v^{k+1} {}_kp_x q_{x+k}$$

$$(DA)_{x:\overset{1}{\overline{n}|}} = \sum_{k=0}^{n-1} (n-k) v^{k+1} {}_kp_x q_{x+k}.$$

*Renty życiowe.*

$$\ddot{a}_x = \sum_{k=0}^{\infty} \ddot{a}_{\overline{k+1}|}\, {}_kp_x q_{x+k} = \sum_{k=0}^{\infty} v^k\, {}_kp_x = \frac{1 - A_x}{d}.$$

$$\ddot{a}_{x:\overline{n}|} = \sum_{k=0}^{n-1} \ddot{a}_{\overline{k+1}|}\, {}_kp_x q_{x+k} = \sum_{k=0}^{n-1} v^k\, {}_kp_x = \frac{1 - A_{x:\overline{n}|}}{d}.$$

$${}_{m|}\ddot{a}_x = \ddot{a}_x - \ddot{a}_{x:\overline{m}|} = \sum_{k=m}^{\infty} v^k\, {}_kp_x = {}_mp_x v^m \ddot{a}_{x+m}.$$

$$\ddot{a}_x^{(m)} = \alpha(m)\ddot{a}_x - \beta(m),$$

gdzie

$$\alpha(m) = \frac{di}{d^{(m)}i^{(m)}} \approx 1, \quad \beta(m) = \frac{i - i^{(m)}}{d^{(m)}i^{(m)}} \approx \frac{m-1}{2m}$$

$$\ddot{a}_{x:\overline{n}|}^{(m)} = \alpha(m)\ddot{a}_{x:\overline{n}|} - \beta(m)(1 - v^n\, {}_np_x).$$

$$(I\ddot{a})_x = \sum_{k=0}^{\infty} (k+1) v^k\, {}_kp_x.$$

*Składki.*

$$P_x = \frac{A_x}{\ddot{a}_x}.$$

$$P^1_{x:\overline{n}|} = \frac{A^1_{x:\overline{n}|}}{\ddot{a}_{x:\overline{n}|}}.$$

$$P_{x:\overset{1}{\overline{n}|}} = \frac{A_{x:\overset{1}{\overline{n}|}}}{\ddot{a}_{x:\overline{n}|}}.$$

$$P_{x:\overline{n}|} = P^1_{x:\overline{n}|} + P_{x:\overset{1}{\overline{n}|}}.$$

$$P_x^{(m)} = \frac{A_x}{\ddot{a}_x^{(m)}}.$$

$$P_{x:\overset{1}{\overline{n}|}}^{(m)} = \frac{A_{x:\overset{1}{\overline{n}|}}}{\ddot{a}_{x:\overline{n}|}^{(m)}}.$$

$$P^{1\ (m)}_{x:\overline{n}|} = \frac{A^1_{x:\overline{n}|}}{\ddot{a}_{x:\overline{n}|}^{(m)}}.$$

$$P_{x:\overline{n}|}^{(m)} = \frac{A_{x:\overline{n}|}}{\ddot{a}_{x:\overline{n}|}^{(m)}}.$$

$$P(\bar{A}_x) = \frac{\bar{A}_x}{\ddot{a}_x}.$$

$$P^{(m)}(\bar{A}_x) = \frac{\bar{A}_x}{\ddot{a}_x^{(m)}}.$$

$$P(\bar{A}^{1}_{x:\overline{n}|}) = \frac{\bar{A}^{1}_{x:\overline{n}|}}{\ddot{a}_{x:\overline{n}|}};$$

$$P(\bar{A}_{x:\overline{n}|}) = \frac{\bar{A}_{x:\overline{n}|}}{\ddot{a}_{x:\overline{n}|}}.$$

$${}_hP_x = \frac{A_x}{\ddot{a}_{x:\overline{h}|}}.$$

$${}_hP^{1}_{x:\overline{n}|} = \frac{A^{1}_{x:\overline{n}|}}{\ddot{a}_{x:\overline{h}|}};$$

$${}_hP_{x:\overline{n}|}^{\ \ 1} = \frac{A_{x:\overline{n}|}^{\ \ 1}}{\ddot{a}_{x:\overline{h}|}};$$

$${}_hP_{x:\overline{n}|} = \frac{A_{x:\overline{n}|}}{\ddot{a}_{x:\overline{h}|}}.$$

*Rezerwy składek.*

$${}_kV_x = A_{x+k} - P_x\ddot{a}_{x+k}, \quad k = 1, 2, \dots .$$

$${}_kV^{1}_{x:\overline{n}|} = A^{\ 1}_{x+k:\overline{n-k}|} - P^{1}_{x:\overline{n}|}\ddot{a}_{x+k:\overline{n-k}|}.$$

$${}_kV_{x:\overline{n}|}^{\ \ 1} = A_{x+k:\overline{n-k}|}^{\ \ \ \ \ \ 1} - P_{x:\overline{n}|}^{\ \ 1}\ddot{a}_{x+k:\overline{n-k}|}.$$

$${}_kV_{x:\overline{n}|} = A_{x+k:\overline{n-k}|} - P_{x:\overline{n}|}\ddot{a}_{x+k:\overline{n-k}|}.$$

*Funkcje komutacyjne.*

$$D_x = v^x l_x,$$

$$C_x = v^{x+1} d_x$$

$$M_x = \sum_{k=0}^{\infty} C_{x+k}$$

$$R_x = \sum_{k=0}^{\infty} M_{x+k}$$

$$N_x = \sum_{k=0}^{\infty} D_{x+k}$$

$$S_x = \sum_{k=0}^{\infty} N_{x+k}$$

$$A_x = \frac{M_x}{D_x}$$
$$A^1_{x:\overline{n}|} = \frac{M_x - M_{x+n}}{D_x}$$
$$A_{x:\overset{1}{\overline{n}|}} = \frac{D_{x+k}}{D_x}$$
$$A_{x:\overline{n}|} = \frac{M_x - M_{x+n} + D_{x+k}}{D_x}$$
$$(IA)_x = \frac{R_x}{D_x}$$
$$(IA)_{x:\overset{1}{\overline{n}|}} = \frac{R_x - R_{x+n} - nM_{x+n}}{D_x}.$$

$$\ddot{a}_x = \frac{N_x}{D_x},$$
$$\ddot{a}_{x:\overline{n}|} = \frac{N_x - N_{x+n}}{D_x},$$
$${}_{n|}\ddot{a}_x = \frac{N_{x+n}}{D_x}$$
$$(I\ddot{a})_x = \frac{S_x}{D_x}.$$

$$P_x = \frac{M_x}{N_x},$$
$$P^1_{x:\overline{n}|} = \frac{M_x - M_{x+n}}{N_x - N_{x+n}},$$
$$P_{x:\overset{1}{\overline{n}|}} = \frac{D_{x+n}}{N_x - N_{x+n}},$$
$$P_{x:\overline{n}|} = \frac{M_x - M_{x+n} + D_{x+n}}{N_x - N_{x+n}}.$$